Problem Set 4, Suggested Answers

5.2 u^i represents the preference ordering $\succsim_i$ if $u^i(x) \geq u^i(y)$ if and only if $x \succsim_i y$. But $v^i(x) = a + bu^i(x) \geq a + bu^i(y) = v^i(y)$ if and only if $u^i(x) \geq u^i(y)$.

5.5. It is a well established mathematical result that a sufficient (not necessary) condition for achieving a maximum of a real-valued continuous function is that the search be over a compact (closed and bounded) domain. The truncation of $B^i(p)$ bounds the set, makes it compact, and hence ensures existence of a well defined maximum. For example let preferences be described by the utility function $u(x,y) = xy+x+y$, let the budget be $\tilde{M}^i(p) = 1000$, $p_x=0$, $p_y=1$. Then $\tilde{D}^i(p) = (\sqrt{c^2 - 1000^2}, 1000)$ but $D^i(p)$ is simply not well defined. For any point (x,y) fulfilling budget constraint proposed for $D^i(p)$ there is another, with a larger value of x, that is preferable. Hence $D^i(p)$ is not well defined even when $\tilde{D}^i(p)$ is well defined.

Chapter 6, Exercises 6.1, 6.2.

6.1. It is an assumption. The assumption unfortunately is not made explicitly but sneaks in as part of the definitions of agent behavior. It does not appear explicitly in axiomatic form. Whenever we write an expression like $\tilde{D}^i(p)$ or $\tilde{S}^j(p)$ as agent optimization subject to the constraints implied by prevailing prices, we are assuming implicitly that prices are treated parametrically.

6.2. Assume P.V (for convenience --- it is not necessary) and use Theorem 4.1. $\tilde{\pi}^j(p) = \max_{y \in Y^j} p \cdot y = p \cdot \tilde{S}^j(p)$ (Starr, p. 96). Recall Theorem 4.1, under the assumptions presented, $\tilde{S}^j(p)$ is well defined and a continuous function of p. So $p \cdot \tilde{S}^j(p)$ is also well defined. Further the dot product is a continuous function of its arguments, so $p \cdot \tilde{S}^j(p)$ is a continuous function of p.